When to use the normal model in place of binomial?

Sometimes it is best to switch gears from doing a binomial model to a normal model. This utilizes an approximation that will be greatly discussed later on. When should we switch from binomial to normal? And once we make that decision, how do we do it? First off, there are a couple things to check before even considering the switch. In the binomial model and before considering the switch, identify n, x, and p. We must have np and $n(1-p)$ both be at least 10. If not, then the normal approximation is just not happening – it's going to be a binomial problem. If these are both 10 *and* you would like to find the probability of a range of many values, then the normal switch is probably good. Consider the following case:

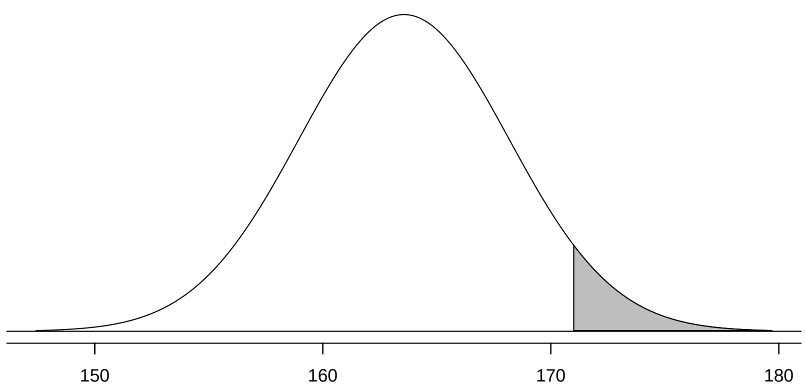
$$P(\text{at least 171 right handers in the class of 188})$$

[This is problem 25 on page 395. $n = 188$, $x = 171$, and $p = 0.87$ for this problem.] Here we are looking for a wide range of values. We could compute this the binomial way if we like:

$$\begin{aligned}
&P(\text{at least 171 right handers in the class of 188})\\
&= P(171 \text{ or } 172 \text{ or } \ldots \text{ or } 188 \text{ RH in a class of } 188)\\
&= P(171 \text{ RH of 188 students}) + \ldots + P(188 \text{ RH of } 188) \qquad (1)
\end{aligned}$$

To actually compute all this would be long and grueling. Instead, we can check that np and $n(1-p)$ are each at least 10 (they are), and since we are computing a wide range of binomial values, we switch to the normal model. We already have the cutoff: $x = 171$. We just need the mean and standard deviation, which are given by formulas: $\mu = np = 188 * 0.87 = 163.56$ and $\sigma = \sqrt{np(1-p)} = \sqrt{1880.87 * 0.13} = 4.61$. These formulas will always be the same; only adjust n, x, and p (which will affect μ and σ). Now this is a "normal problem".

We set up the normal plot using μ, σ, and x, and also compute the Z-score.



$$Z = \frac{171 - 163.56}{4.61} = 1.61$$

From here, use the Z-table to find the proportion of the area that lies above $Z = 1.61$: 0.053. If we wanted the exact value, we would have to do it the binomial way (equation (1)) but 0.053 is a good approximation.

We could have more than one cutoff, like if we wanted to find the chance of 171 to 178 right handed students, and this again would be okay to do as a normal problem with the same μ and σ.