Section 2A: Discussion
November 5th, 2009

The first two sections should help in preparation for hypothesis tests. The last section has some sample problems for what was covered over the last week.

1 Innocent or guilty?

In a court of law, the defendant is assumed innocent unless the evidence overwhelmingly implies she is guilty.

- What are the courts null and alternative hypotheses?
- What is the "mechanism" to reject the null hypothesis in the court?
- What two types of errors can be made?
- Is there a way for the court to always make a correct decision?

		Reality	
		H_0 TRUE	H_A TRUE
Test	Do not reject H_0		
Conclusion	Reject H_0		

2 A new treatment

The FDA wants to examine a generic drug to treat skin inflammation to compete with a name-brand drug. They test the drug's efficacy by doing a clinical trial where patients are randomly assigned to the generic and name-brand drugs.

- What should be the FDA's hypotheses setup?
- What errors can the FDA make?

In examining the results, the FDA only cares about if the drug did or did not cause the inflammation to go down. In the generic group, 507 of the 800 were deemed successful while 452 of the 700 in the name-brand group were said to be successful. What summary statistics should we compute for each group if we want to compare the two?

3 Examples

The problems below have all been modified from the book.

p485, #53. IQs of East State University's students can be described as $N(130, 8)$. IQs from West State University can be described as $N(120, 10)$.
(a) We select a student at random from East State. What is the probability this student's IQ is above 125?

First draw the picture (area above 125 of a normal distribution centered at 130). Then compute $Z = \frac{125-130}{8}$. Look up Z in the normal table to find the lower tail probability (0.27). Since we want the upper tail, take one minus the value: $1 - 0.27 = 0.73$.

(b) We select 3 random East State students and consider their mean IQ, $\bar{x}$. What is the distribution of $\bar{x}$? What is the probability this mean is above 125?

Since the individual observations are normally distributed, the mean will be normally distributed. The sample mean will follow $N(130, \sigma_{\bar{x}} = 8/\sqrt{3})$. Now we just have the same normal problem as (a) but with a different standard deviation.

(c) Would we still be able to complete parts (a) and (b) if the IQ scores were not approximately normal?

Not without substantially more information. In particular, the sample size in part (b) is too small to ensure the sample mean would be normally distributed *if* the individual observations were not already nearly normal.

(d) Complete parts (a) and (b) for students from West State.

I will omit this answer.

p506, #22. A national health organization warns that 30% of middle school students have been drunk. A local health agency randomly and anonymously surveys 110 of the 1212 middle school students in its city and finds only 21 of them report having been drunk.
(a) What proportion of students in the sample have been drunk?

$\hat{p} = 21/110 = 0.191$.

(b) Does this mean the city's youth are not drinking as much as the national health organization warned?

Not necessarily. It is unclear whether natural variation in the sample may result in only 19.1% in the sample.

(c) Can we assume our estimate from (a) approximately follows a normal distribution? If so, with what mean and standard deviation?

The sample was random (e.g. observations are independent), the sample size was under 10% of the population, and we have at least 10 successes and failures. (Yes, a normal model fits.)

IF we suppose the true proportion is 0.3, then it would follow $N(0.3, \sqrt{0.3 * (1 - 0.3)/110})$. This question could have been worded better.

(d) Create a 95% confidence interval for the proportion of the city's middle school students who have been drunk.

We will use $SE = \sqrt{0.191 * (1 - 0.191)/110} = 0.037$. Then our confidence interval, using $Z^* = 1.96$, can be computed via $\hat{p} \pm 1.96 * 0.037$: $(0.118, 0.264)$. We are 95% confident the true proportion of local middle school students who have been drunk is in this interval.

(e) Based on your answer to (d), does 30% seem to be a plausible proportion of middle school students who have been drunk for the local area?

It does not seem plausible since it is not in our confidence interval.

The p-value is defined to be the probability of the result or a more extreme result IF the null hypothesis were actually true.

p527, #3. Pollster's checked a governor's approval rating since his involvement in a recent scandal. They test the hypothesis that there is no change from previous polls (61%) with the alternative that his approval has dropped based on the current poll (57%), and they found a p-value of 0.24. Which conclusion is appropriate?

(a) If his approval dropped, it dropped by 24%.
(b) There is a 24% chance his approval did not drop.
(c) If there was no real change in approval, the chance of observing a 4% or larger drop is 24%.
(d) If there was a drop in his approval, 24% of the time we would find the difference.

The correct answer is (c), which closely describes what a p-value represents.